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Kapteyn, Trigonometrical Formulæ), presented by the Laboratory; G. W. Hill, Collected Mathematical Works, vol. iii., presented by the Author; Oxford University Observatory, Miscellaneous Papers, vol. ii., presented by the Savilian Professor of Astronomy; Radcliffe Catalogue of Stars for 1900 (A. A. Rambaut), presented by the Radcliffe Trustees; Eighteen Charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich; and Ten Charts presented by the San Fernando Observatory.

On the Ancient Eclipses of the Sun. By E. Nevill.

It was with much interest that I read Mr. Cowell's recent papers on the ancient eclipses of the Sun and the values they indicate for the secular accelerations of the longitudes of the Sun, Moon, and lunar node, and it is with some curiosity that I have waited for the last six months to see if any of the distinguished astronomers conversant with the subject would critically examine the results which Mr. Cowell has deduced.

In the first of these papers (*Monthly Notices*, vol. lxv. p. 861) Mr. Cowell reaches the conclusion that if the Moon's secular acceleration be taken as $+7''{\cdot}0$, in order to represent the ancient eclipses of the Sun it is necessary to reduce to about $+4''{\cdot}4$ the secular acceleration of the Moon's angular distance from its node, corresponding to a reduction of the secular acceleration of the Moon's node to only a third of its theoretical value.

In the second of these papers (*Monthly Notices*, vol. lxvi. p. 3) Mr. Cowell points out that, as this reduction of the Moon's nodical secular acceleration is inconsistent with the theory of gravitation, the requisite value of the secular acceleration of the Moon's angular distance from its node should be obtained by adopting the theoretical value of the secular acceleration of the node and increasing the secular acceleration of the Moon's mean longitude from $+6''{\cdot}8$ to $+10''{\cdot}9$, and ascribing a secular acceleration of $+4''{\cdot}1$ to the mean longitude of the Earth, so as to leave unchanged the epoch of conjunction of the Sun and Moon in mean longitude.

The results deduced in these two papers are quite distinct, and form two questions which may well be examined separately.

The first is the existence of an apparent secular acceleration of $+4''{\cdot}4$ in the angular distance of the Moon from its node.

The records of the ancient eclipses of the Sun and the values they indicate for the secular accelerations of the Moon's mean longitude, longitude of node, and longitude of perigee have been examined in some detail in one of the sections of my unpublished memoir on the errors of the lunar tables, and from it can be derived further information on the subject considered in Mr. Cowell's papers.

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The eclipses are referred to Hansen's *Tables du Soleil* and to Hansen's *Tables de la Lune* under the following forms :—

A. Hansen's Tables, with his final values for the secular accelerations and motion of the node.

B. Hansen's Tables, after replacing his tabular values of the secular accelerations by those derived theoretically from the secular diminution of the eccentricity of the terrestrial orbit and making the small consequent corrections necessary to bring the amended tables into accord with the modern observations.

C. Hansen's Tables, after applying the corrections indicated by a complete discussion of the lunar observations between the years 1650 and 1900, employing for the secular acceleration of the mean longitude the value $+7''\cdot43$, of the longitude of the node the value $+6''\cdot07$, and of the longitude of the perigee the value $-32''\cdot53$.

The only eclipses considered are those at least two thousand years prior to the present epoch, as those of later date are of little real value for the desired purpose.

In the following $\Delta\phi$ represents the distance of the central line of totality from the assumed place of observation, measured on the meridian in degrees of latitude, a positive sign showing that the central line of totality is to the south of the assumed place of observation.

I. *The Eclipse of Agathocles.*—Recorded as having happened on -309 August 14, on the second day after Agathocles had left Syracuse on his way by sea towards Carthage. It being unknown whether he sailed south or north of Sicily, it is uncertain whether the eclipse was observed from 36½° or from 38° N. latitude, in both cases the longitude being about 15° E. The different modifications of the lunar tables yield the following values of $\Delta\phi$ measured from the latitude of Syracuse (37° N.), and to them has been now added the value that has been obtained from Mr. Cowell's data by an approximate calculation:

$$A = +1^{\circ}\cdot8 \qquad B = +0^{\circ}\cdot8 \qquad C = +0^{\circ}\cdot8 \qquad \text{Cowell} = -0^{\circ}\cdot9$$

As the width of the zone of totality exceeds two degrees, it is apparent that the eclipse is represented by all four systems of data.

II. *The Eclipse of Pelopidas.*—A very large, but not necessarily total, eclipse of the Sun, said by Plutarch to have occurred before Pelopidas left Thebes on the expedition against Alexander of Pheræ. Apparently it must have been the eclipse of -363 July 13. The values of $\Delta\phi$ are

$$A = -10^{\circ}\cdot8 \qquad B = -11^{\circ}\cdot8 \qquad C = -11^{\circ}\cdot5 \qquad \text{Cowell} = -16^{\circ}\cdot0$$

These all indicate a large eclipse, but the results are of little use.

III. *The Eclipse of Ennius.*—Stated by Cicero to have occurred at Rome in -399 June 21 toward sunset. It appears doubtful if the Sun was totally eclipsed before sunset, or even if

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it were total at all. If there had been any certainty on this point it would have been a most valuable criterion, and deserved the great weight assigned it by Hansen. The values of $\Delta\phi$ are

$A = -1^\circ\!\cdot2$ $B = +10^\circ\!\cdot4$ $C = +7^\circ\!\cdot4$ Cowell $= +3^\circ\!\cdot1$

According to Hansen's values the Sun set after totality; but according to the modified forms it must have set some time before totality, and the eclipse cannot have been total near Rome.

IV. *The Eclipse of Lysander.*—Stated by Xenophon to have occurred in the autumn of the year that Lysander took Athens, which would correspond to the eclipse of —403 September 3. It must have been a large eclipse, but not necessarily a total eclipse in Greece. The values of $\Delta\phi$ are

$A = -0^\circ\!\cdot2$ $B = +2^\circ\!\cdot6$ $C = +1^\circ\!\cdot6$ Cowell $= -2^\circ\!\cdot4$

The eclipse was annular, and according to all four sets of data must have been very large at Athens.

V. *The Eclipse of Thucydides.*—Stated by Thucydides to have occurred in the summer of the first year of the Peloponnesian War, and to have been nearly, but not quite, total at some spot in Greece—probably Athens, but quite possibly some place in the northern Ægean Sea, or even in the Ionian. It is this uncertainty which weakens the value of this eclipse as a test. The eclipse was only annular, and occurred so soon before sunset that stars might have been seen as recorded, even had the eclipse been only a large eclipse. The values of $\Delta\phi$ are

$A = -17^\circ\!\cdot8$ $B = -1^\circ\!\cdot8$ $C = -5^\circ\!\cdot8$ Cowell $= +0^\circ\!\cdot1$

Hansen's values fail to account for this eclipse; both the theoretical secular accelerations and Mr. Cowell's data represent it as seen at Athens, and the data C as if seen from the northern Ægean.

VI. *The Eclipse of Thales.*—Stated by Herodotus to have occurred during a battle, fought probably at some place in Asia Minor, between the Lydians under Alyattes and the Medes under Cyaxares, at the end of a war of some length, during the period when Labynetus was king of Babylon. This war must have happened after the destruction of the Assyrian Empire by the Medes and Babylonians, and so have been later than the year —600. According to both Herodotus and Pliny, it was the eclipse predicted to the Ionians by Thales, and the date assigned by Pliny corresponds with that of —584 May 28.

There exist no means of ascertaining the position of the battlefield, and all that can be assured is that in all probability it was some spot on the great road from the Cilician Gates, through Iconium, to Sardis; though there is always the possibility that it was on the northern road by which Cyrus subsequently invaded Lydia, which passed by Pteria, in Northern Cappadocia.

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Airy, Hind, and Hansen identify this eclipse with that of −584 May 28, which, according to Hansen's Tables, crossed Southern Asia Minor; and it is difficult to reconcile the existing accounts with any other eclipse. It is true that recent investigation has shown that the data assumed in Hansen's Tables require such modification that the zone of totality of the eclipse of −609 September 28 may have passed over the north of Asia Minor, crossing the northern route afterwards followed by Cyrus; but as this eclipse happened before the fall of the Assyrian Empire it must have occurred prior to the war between the Medes and Lydians. Similar permissible changes in the data of Hansen's Tables show that the zone of totality of the eclipse of −556 May 17 may also have crossed the southern portion of Asia Minor, traversing the southern route from the Cilician Gates to Sardis, upon which the battle was fought in all probability; but if this eclipse be assumed, Crœsus could not have succeeded Alyattes until after −556, and it is difficult to compress the events of his reign into the ten years between that date and the capture of Sardis by Cyrus in about −545. Yet it is difficult to reconcile the earlier date of the eclipse with the statement of Herodotus that Labynetus was king of Babylon both at the epoch of the eclipse and at the capture of Sardis by Cyrus, as Nabu-kudur-uzur (Nabuchadrezzar II.), who reigned in Babylon in −584, died in −561; but if the later eclipse were adopted, then the king of Babylon at both events may have been Nabu-nahid (Nabonidus), who reigned from −556 to −537.

The values of $\Delta\phi$ for the eclipse of −584 May 28, measuring from Iconium, are

$A = -2^\circ\cdot 0 \qquad B = +3^\circ\cdot 1 \qquad C = +1^\circ\cdot 1 \qquad \text{Cowell} = -3^\circ\cdot 0$

and as the zone of totality considerably exceeded two degrees in width, all four sets of data will serve to bring the phase of totality within permissible limits of latitude. But according to the different data, totality is reached at sunset at east longitudes:

$A = 44^\circ \qquad B = 23^\circ \qquad C = 29^\circ \qquad \text{Cowell} = 29^\circ$

and as the probable longitude of the battlefield lies between 30° and 32°, except in the case of Hansen's data, the Sun must have set before the eclipse was total, though in all cases it would have been very largely eclipsed before reaching the western horizon.

Would this suffice to meet the exigencies of the account given by Herodotus?

The values of $\Delta\phi$ for the eclipse of −556 May 19 are

$A = -5^\circ\cdot 0 \qquad B = +2^\circ\cdot 7 \qquad C = +1^\circ\cdot 0 \qquad \text{Cowell} = +5^\circ\cdot 0$

and the zone of totality is narrow, little exceeding half a degree. The Sun in all cases would at totality be well above the western

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horizon ; and according to data C, and possibly B, it would cross the probable position of the field of battle. Apart from chronological difficulties, this eclipse seems to agree with the account given by Herodotus even better than the earlier one. To throw over the circumstantial account given by Herodotus on the ground that he was credulous and related fables does not seem sound ; and it must be remembered that recent discoveries have generally confirmed the accuracy of Herodotus, even when his accounts seemed most improbable to earlier critics.

VII. *The Eclipse of Larissa.*—According to Xenophon this eclipse occurred at the capture of the "Median" city of Larissa by the "Persians" ; and, from the account of its effects, it must have been a total eclipse to have spread such consternation amongst the defenders of the city that they fled from the walls. The ruins of Larissa seen by Xenophon have been identified with certainty with the mounds of Nimrud, covering the remains of the Assyrian city of Calah. As the Sun represented Asshur, the ruling deity of Assyria, its eclipse would account for the terror of the defenders of Calah, who would take the darkness overspreading the Sun as indicating that the gods had decreed the ruin of Assyria.

With the imperfect knowledge of Assyrian chronology then alone available to guide him, Airy was led, in 1856, to identify this eclipse with that of −556 May 19, and showed that, according to Hansen's Tables, the narrow zone of totality passed nearly centrally over Larissa, and that there was no other eclipse within a period of forty years which could have been total at Larissa. But this eclipse cannot be reconciled with the theoretical value of the secular acceleration of the Moon's longitude, and it has proved the great obstacle to any reduction of Hansen's tabular values of the accelerations to the smaller values indicated by modern observations, as well as by theory, because any such reduction must throw the zone of totality of this eclipse many hundreds of miles to the south of Larissa.

Professor Newcomb and others seem disposed to evade the difficulty by either ignoring the eclipse, throwing over the tradition as unfounded, or ascribing the cause to some extraordinary meteorological phenomenon, and not to an eclipse of the Sun. I do not think this can be justified, and can see no reason for disbelieving the traditional account, which is so much in harmony with what is known of the Assyrian character ; all the more so as the whole course of recent discovery has tended to confirm the accuracy of these traditional records of the events of ancient history.

As the eclipse of −556 May 19 cannot be reconciled with the value of the secular acceleration indicated both by theory and by the modern observations of the Moon, it is certain that it cannot be the eclipse which occurred at the capture of Larissa.

Since the epoch of Airy's identification of this eclipse much progress has been made in our knowledge of Assyrian history

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and chronology, and the actual dates of the greater part of the known history of Assyria have been determined with certainty; but, often as this eclipse has been discussed since Airy's memoir of 1857, it does not seem to have been noticed that the capture of Larissa, the Assyrian city of Calah, by either Medes or Persians cannot possibly have taken place in —556. Nineveh, Calah, and the other great cities of Assyria disappear from history before —600, and no reference to the existence of these great cities is known to occur at any subsequent period, either in Babylonian, Persian, Egyptian, or Grecian records. They were all destroyed together during the great invasion prior to —600, and the very state of the ruins at the present time shows that the destruction was sudden and once for all. Every inhabitant perished or was transported in slavery to some distant spot, and the cities were never allowed to be re-inhabited. The capture of Larissa to which the tradition refers must be the capture of the Assyrian Calah by the Medes and Babylonians prior to —600, and could not refer to the capture by the Persians of a town built by the Medes on the site of the older Assyrian city of Calah, a town of whose existence there is no record, and of whose remains there is no sign amongst the existing ruins of Calah. As before remarked, the very state of the ruins shows that Calah was an Assyrian city when destroyed prior to —600, and that its destruction was final.*

The great siege of Nineveh began in — 608, and after a long siege ended with the destruction of the city in —605. Were the surrounding cities of Calah, Dur-Sargina, &c., besieged and captured before or after the capital, Nineveh? The whole tenor of ancient history shows that they would be besieged and taken one after the other, and not all at once. If Calah was besieged and taken before Nineveh, then the eclipse must have been that of —609 September 30. If besieged and captured after Nineveh had fallen, then the eclipse must have been that of —602 May 18.†

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* Even if it had been permissible to suppose that the Medes, after capturing and destroying the city of Calah, had refounded a Median town on the site, though a supposition inconsistent both with history and with the state of the ruins, and that it was the capture by the Persians of this hypothetical new town of the Medes, to which the tradition referred, still even this would not save the eclipse of —556 May 19. For just as it is certain that the capture of the Assyrian city of Calah by the Medes and Babylonians occurred nearly half a century before this eclipse of —556 May 19, so is it equally certain that Cyrus, King of Ansan and Persia did not revolt against the Medes until the sixth year of Nabu-nahid (Nabonidus), King of Babylon, which corresponds with —548, eight years after the eclipse, and did not attack this region until the invasion of Babylonia in —538.

† It is known that the great palace of Calah was burnt down from some cause and rebuilt by Assur-bani-pal's successor, Assur-etil-ilani, some time between —620 and —615. It has been assumed that this burning took place during the Scythian (or Mandaian) invasion, which occurred about the time of Assur-bani-pal's death in —625. This assumed capture of Calah, and of Calah only, out of all the surrounding cities by the Scythians about —625 does not

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The values of $\Delta\phi$ for these two eclipses are :

−602 May 18	A = +12°·0	B = +0°·2	C = +2°·0	Cowell = −0°·6
−609 Sept. 30	= − 9·7	= −1·2	= −3·4	= −8·9

The zone of totality of these eclipses was wide in each case, and both occurred near midday. Neither eclipse can have been total at Calah, according to Hansen's data ; but the eclipse of −602 May 18 was total on all three other systems of data, and the eclipse of −609 September 30 was total according to the theoretical values of the secular acceleration. This great obstacle to the adoption of the theoretical value of the secular accelerations vanishes.

VIII. *The Eclipse of Archilochus.*—Described by Archilochus in one of his poems, and from the description must have occurred at midday, and been at least annular, but probably total, where it was seen. Beyond this everything is vague. It may have been any eclipse of this nature between −710 and −640 ; and, though probably observed at Thasos, may have been seen from Paros, or even from some other place on the Ægean Sea. The eclipses during this period which it might have been are those occurring on

−710	March	14	Total
−703	October	19	Annular
−688	January	11	Annular
−656	April	15	Total
−650	June	7	Annular
−647	April	6	Total
−640	November	11	Annular

Mr. Cowell, like Professor Millosevich, selects the eclipse of −647 April 6 because it agrees with the data he adopts ; but this is considerably later than the date usually assigned to this

harmonise with the rest of the events of this period, nor even with probability. The very fact that an attempt was made to rebuild the palace, though in an inferior style, by Assur-bani-pal's successor, Assur-etil-ilani, would strongly indicate that the palace was destroyed by an accidental fire, similar to that which happened nearly three centuries before. If the city had been captured and burnt with the palace at this earlier epoch, it is not likely that an attempt would have been made to re-erect an inferior building amongst the ruins of a destroyed city when the king had so many other magnificent palaces in the surrounding cities. But if the palace of the second city of Assyria had been accidentally burnt, leaving the great city in existence, then it is probable enough that an attempt should have been made to restore the palace accidentally destroyed, though even in an inferior style, rather than leave the ancient capital of Assyria without any residence for the king. Certainly no attempt was made to restore any of the other palaces when once they had been destroyed with the surrounding cities during the conquest by the Medes and Babylonians.

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poet. Still, as pointed out by Mr. Lynn, Archilochus has referred to the capture of Magnesia by the Cimmerians, which took place during the reign of Assur-bani-pal of Assyria, and so must be later than —668, but probably before —660, the beginning of the trouble with Elam. The values of $\Delta\phi$ for these eclipses, supposed seen from Thasos, are :

−710	March	14	A = + 8°·1	B = −10°·7	C = − 8°·6	Cowell = ...
−703	Oct.	19	− 7·3	+ 2·7	− 0·7	
−688	Jan.	11	+13·6	+ 6·3	+ 7·9	−1·8
−656	April	15	+16·1	− 2·0	+ 0·7	
−650	June	7	− 8·8	− 8·2	− 8·2	
−647	April	6	+ 0·3	−12·9	− 9·6	+0·5
−640	Nov.	11	−25·4	− 7·0	−12·5	

From this it appears that the eclipse of Archilochus may be represented on the data A by assuming it to be the eclipse of —647 April 6, as seen from Thasos ; on the data B, employing the theoretical accelerations, by assuming it to be the eclipse of —656 April 15 as seen from Thasos or the eclipse of —703 October 19 as seen from Paros ; on the data C by assuming it to have been either the eclipse of —703 October 19 or that of —656 April 15, as seen from Thasos ; or by Mr. Cowell's data by supposing, as he does, that it was the eclipse of —647 April 6, as seen from Thasos, or the eclipse of —688 January 11.

It is apparent that this eclipse can afford no information as to the true values of the secular accelerations, as none of these seven eclipses, seen from either Thasos or Paros, can be found which will harmonise with any permissible values.

IX. *The Eclipse of Susa.*—Recorded in a tablet of the reign of Assur-bani-pal, immediately before his expedition against Elam in —659 : "Te-umman devised evil, and Sin devised against him forebodings of evil. In Tammuz an eclipse at evening. He troubled the lord of light, and the setting Sun thus also for three days was troubled. It went forth for the end of the reign of Elam." An eclipse of the Sun could not trouble the Sun for three days, but Dr. Pinches considers that the eclipse and the troubling of the Sun should be regarded as distinct occurrences. Even were there objections to so differencing them, the record may be interpreted as reading that the Sun was so troubled or eclipsed "*on*" three days, or that there had been three eclipses, probably at sunset.

The inscription does not say that the eclipse was total ; a large eclipse at sunset would suffice ; but the month, Tammuz, of the specified eclipse is only consistent with the eclipse of —660 June 27. The values of $\Delta\phi$ are :

A = + 1°·6 B = + 13°·5 C = + 7°·9 Cowell = + 6°·0

All occurred near sunset. According to Hansen's data it would have been annular before sunset ; but, according to the data C

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and those adopted by Mr. Cowell, only a large partial eclipse at sunset, whilst the system depending on the purely theoretical data would make it a much smaller eclipse. There are two other eclipses at this time; one, that of −661 January 12, would have been a large eclipse at sunset; and another, in −663, would also have been visible, but not at sunset.

X. *The Eclipse of Esar-haddon.*—Recorded in the form, "Since my lord went to Egypt an eclipse has taken place in the month of Tammuz" (June), on an Assyrian tablet of the reign of Esar-haddon, between −680 and −668. The only large eclipse which can answer to this description is that which occurred on −678 June 17. There is absolutely no other in the reign of this king which it could have been. Yet this eclipse occurred several years before Esar-haddon's great Egyptian wars; and, if it be the eclipse of this date, the statement that it occurred when the king had gone to Egypt must refer to one of the early expeditions to the frontiers of the empire, which are only imperfectly recorded in the early annals of this king of Assyria.

The values of $\Delta\phi$ are:

$A = +19^\circ\!.6 \qquad B = +5^\circ\!.6 \qquad C = +11^\circ\!.3 \qquad \text{Cowell} = +4^\circ\!.8$

It will be seen that this eclipse would have been a very large eclipse at Nineveh according to the data employed by Mr. Cowell, or those depending on the theoretical secular accelerations.

XI. *The Eclipse of Nineveh.*—Recorded under the form "Eponomy of Bur-Sagall, Governor of Gozan. A revolt in Assur took place in the month Sivan, and the Sun was eclipsed" on an Assyrian cuneiform tablet as occurring in the eighth year of Assur-dan III., corresponding to −762. Hence the eclipse must have been that of −762 June 15. It will be seen that the eclipse is not stated to have been total, nor is anything recorded indicating that it was total; and though undoubtedly it must have been large, it must not be assumed to be necessarily a total eclipse, as assumed by Mr. Cowell. In consequence it is not of great value for ascertaining the probable amount of the secular accelerations, though highly important as fixing Assyrian chronology. The values of $\Delta\phi$ are:

$A = -4^\circ\!.1 \qquad B = -6^\circ\!.7 \qquad C = -5^\circ\!.9 \qquad \text{Cowell} = -0^\circ\!.8$

As the zone of totality was fairly wide, Mr. Cowell's data make it total at Nineveh, but on the other systems it is only very large. Either would satisfy the record.

These eclipses are considered in my memoir, and to them may be added the new eclipse recorded in the Babylonian tablet deciphered by Mr. King and quoted in part by Mr. Cowell.

XII. *The Eclipse of Babylon.*—Recorded as occurring on the "26th day of the month Sivan in the 7th year." From the other details the eclipse was most likely a total eclipse with a vivid

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corona. Assuming that the eclipse occurred in the first half of the eleventh century it would seem certain that this eclipse must have been that of −1069 June 19, which occurred in the month Sivan of the seventh year of E-Ulmas-sakin-sumi, King of Babylon, thus being in strict accord with the record. But as this eclipse could not possibly have been total at Babylon according to his adopted data, Mr. Cowell is led to substitute the eclipse of −1062 July 30. The discrepancy of seven years in the epoch of the king's reign might not be deemed fatal to this identification, owing to the uncertainty in these very early epochs; but can July 30 ever have been in the month Sivan, which corresponds with May or June? Could it even have corresponded with Tammuz, the next month, or must it not be held to be some day of the month after? As the inscription must be regarded as an actual record, the month cannot have been in error, and the record cannot be changed by reading Tammuz, or even some later month, for the Sivan actually recorded.

This eclipse of −1069 June 19 is the only one which occurred during the month Sivan which could have been visible as a total eclipse from Babylon during the first half of the eleventh century. There are annular eclipses in this month during this period, but they could not have been even large eclipses near Babylon, and there are other eclipses which might have been total near Babylon, but could never have been described as occurring in the month Sivan.

If it were permissible to assume an earlier epoch, then the record might refer to one of the kings of the Isin dynasty, and correspond with either the eclipse of −1116 June 28 or −1123 May 18, both of which may have been total at Babylon.

The values of $\Delta\phi$ for the preceding eclipses are:

−1062 July 30	A = − 2°·2	B = − 7°·0	C = − 5°·6	Cowell = + 0°·8
−1069 June 19	= +30·7	= +13·0	= +16·5	= +12·3
−1116 June 28	= +10·8	= − 6·9	= − 0·3	= + 4·0
−1123 May 18	= +11·7	= + 1·2	= + 3·2	= − 2·1

It will be seen that the line of totality of the eclipse of −1069 June 19 cannot be brought over Babylon by any of the four systems of data or by any system within the limits of possibility. Hence, though otherwise in complete accordance with the recorded account, it cannot have been the recorded eclipse. The eclipse of −1062 July 20 can be made total at Babylon either by Mr. Cowell's data or by a slight modification of the very different data obtained by Hansen, and therefore by any number of intermediate combinations lying between the two systems. If it be adopted as the eclipse corresponding to that recorded in the cuneiform tablet, it must be held as of little use for indicating the true value of the secular acceleration; but the discordance between the month of the eclipse and that stated in the inscription must be held to render such an identification very doubtful.

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Mr. Cowell does not give any reasons for assigning this eclipse to a period subsequent to −1100, nor does he state whether Mr. King furnished such reasons, nor even whether the rest of the inscription (by stating the king's name or dynasty) affords means for fixing such a limit. Unless such reasons exist for dating the record after −1100, the eclipse would seem more likely to have been that of −1116 June 28, or −1123 May 18, than that of −1062 July 30, since the eclipse of −1069 June 19 is out of the question, as not even large near Babylon. If the inscription permits of the recorded eclipse being identified with one of these earlier eclipses it would be most valuable for fixing the true value of the secular acceleration and motion of the lunar node, as it would clearly distinguish between the four representative sets of data.

The preceding comparisons show that the eclipses, which by slight permissible alterations in the assumed data may be represented by the four very different sets of assumed values for the secular accelerations and motions of the Moon's node, are as follows ;—

Name.	Obs. Phenomenon.	A		B		C		Cowell.	
		Date.	Δφ.	Date.	Δφ.	Date.	Δφ.	Date.	Δφ.
I. Eclipse of Agathocles	Total	− 309	+1°·8	− 309	+0°·8	− 309	+0°·8	− 309	−0°·
III. Eclipse of Ennius ...	Large	− 399	−1·2	...	...	...	...	− 399	+3·
IV. Eclipse of Lysander	Large	− 403	−0·2	− 403	+2·6	− 403	+1·6	− 403	−2·
V. Eclipse of Thucydides	Nearly total	...	...	− 430	−1·8	− 430	−2·8*	− 430	+0·
VI. Eclipse of Thales ...	Total	− 584	−2·0	...	...	− 556	+1·0	...	...
						− 584	+1·1	...	...
VII. Eclipse of Larissa ...	Total	− 556?	−0·6	− 602	+0·2	− 602	+2·0	− 602	−0·
VIII. Eclipse of Archilochus	Total	− 647	+0·3	− 656	−2·0	− 656	−0·7	− 647	+0·
				− 703	+2·7	− 703	+0·7	− 688	−1·
IX. Eclipse of Susa ...	Large	− 660	+1·6	...	...	− 660	+7·9	− 660	+6·
X. Eclipse of Esarhaddon	Large	...	...	− 678	+5·6	...	...	− 678	+4·
XI. Eclipse of Nineveh...	Large	− 762	−4·1	− 762	−6·7	− 762	−5·9	− 762	−0·
XII. Eclipse of Babylon	Total	−1062	−2·2	−1123	+1·2	−1116	−0·3	−1062	+0·
								−1123	−2·

Mr. Cowell considers that the fact that the system of secular mean motions and accelerations adopted by him renders total the five eclipses

−1062	July	30	at	Babylon
− 762	June	14	„	Nineveh
− 647	April	5	„	Thasos
− 430	August	3	„	Athens
+ 197	June	3	„	Utica

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* Observed from the northern Ægean instead of Athens.

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is so remarkable that it cannot be held a chance coincidence, but must be regarded as demonstrating the accuracy of the adopted system. It is necessary to remember, however, in considering this claim—

That the eclipse at Babylon is stated explicitly to have occurred in the month Sivan, whereas the eclipse adopted occurred in Tammuz or later;

that the eclipse at Nineveh is not recorded as being total, as assumed without authority by Mr. Cowell, and so may have been only a large eclipse;

that a total eclipse may be found to represent that at Thasos on any system of data, so that it is valueless as evidence of the accuracy of any system;

that the eclipse at Athens, though annular according to Mr. Cowell's data, is only recorded as having been very large, whilst it is represented just as well by other systems of data as by that adopted by him; and

that the eclipse of Utica has not been considered, as its epoch is so late that it cannot be regarded as of any value.

Taking these considerations into account it will be seen that the fact that these five eclipses selected by Mr. Cowell are rendered total by his data cannot be regarded as establishing the accuracy of the data adopted. Nor can it be urged that, though each of the preceding five eclipses may be represented just as well by other systems of data, yet Mr. Cowell's is the only system which will represent five eclipses, for the preceding examination shows that out of the twelve eclipses considered Hansen's theoretical values can be made to represent seven or possibly nine; that the strictly theoretical values of the secular accelerations will represent seven or eight; that the data derived exclusively from the modern observations will represent six at least, but possibly eight; and that Mr. Cowell's data with some slight amendment may be made to represent as many as ten eclipses. The mere fact that a system of data represents a certain number of rather vaguely defined eclipses cannot be regarded as establishing anything.

In truth, the results which have been given show that unless it can be established that an eclipse was to all intents total at a specified place within a narrow definite period of years, it can be of small use as a criterion of the true values of the mean motions and secular accelerations. Of those in the preceding list there is but one possessing these requirements, and that is the eclipse of Larissa, though possibly the eclipse of Babylon may prove to be another.

In my memoir, in addition to the series of eclipses which has been considered in the preceding, there was also taken into account a series of twelve eclipses of the Sun, observed during the Chow dynasty in China, and presumably observed from the

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ancient capital of Heeng-Yang, in Shensee, where there still exists the remains of a very ancient observatory. It is possible, however, that in some cases the place of observation may have been the neighbouring city of Chang-gan, which was the residence of some of the emperors during this period. Some of these eclipses are expressly recorded as total; and all seem to have been very large eclipses, if not at least annular. In each case the exact date of the eclipse is definitely fixed by the record, so that there is no uncertainty in this respect.

The eclipses are :—

				A	B	C	Cowell.
XIII.	−497	Sept. 22	Annular $\Delta\phi =$	− 4°·2	= +3°·6	= +1°·8	= − 4°·2
XIV.	−526	April 18	Annular	+ 8·3	−3·6	+0·0	− 5·4
XV.	−545	Oct. 3	Total	− 6·0	+4·2	+2·9	+ 6·8
XVI.	−548	June 19	Total	+ 4·2	+3·4	+3·3	− 0·2
XVII.	−549	Jan. 5	Annular	+13·2	+1·5	+6·4	+11·7
XVIII.	−574	May 9	Total	− 1·1	−1·9	−1·4	+ 2·2
XIX.	−600	Sept. 20	Total	−20·1	−2·3	−5·1	+ 2·0
XX.	−625	Feb. 3	Annular	+ 6·9	−0·3	+2·4	+ 8·6
XXI.	−663	Aug. 28	Total	−14·8	−3·4	−5·4	−12·2
XXII.	−667	Nov. 10	Annular	−12·3	−0·9	−4·7	−11·7
XXIII.	−668	May 27	Annular	− 3·5	−7·3	−5·5	+ 0·3
XXIV.	−708	July 17	Total	−19·9	−4·2	−5·8	− 3·0

There are several other eclipses during this period recorded as having occurred, whose place of observation is too uncertain to render them of value.

Use was also made of four more recent eclipses, recorded in more indefinite language, only the year and season being stated, but all explicitly described as total. These are the eclipses :—

XXV.	−180	Mar. 4	Total $\Delta\phi =$	+1°·6	= − 3°·5	= − 2°·1	= + 0°·9
XXVI.	−187	July 17	Total	−8·3	+ 1·1	− 1·0	+ 1·9
XXVII.	−299	July 26	Total	−2·9	− 6·9	− 5·9	− 3·3
XXVIII.	−381	July 13	Total	−6·0	−13·8	−11·2	−15·6

This last eclipse is discordant on all systems of data, and could only have been total in Mongolia.

It will be seen that out of these sixteen eclipses observed in China, by regarding those for which $\Delta\phi$ is less than $2\frac{1}{2}^\circ$ as total or annular, and those for which $\Delta\phi$ lies between $2\frac{1}{2}$ and 6° as so large and conspicuous as to be practically total, the number in the case of each of the four sets of data being considered is

	A	B	C	Cowell.
Total or annular	2	6	6	6
Very large	5	7	8	4
Discordant	5	1	1	4

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Practically the last three systems all serve to represent these Chinese eclipses as well as could be anticipated, and show the caution that must be employed in drawing deductions from such a circumstance.

In concluding this examination it must be pointed out that the preceding considerations do not invalidate Mr. Cowell's conclusion that the Moon's argument of latitude F possesses a secular acceleration of about $+4''{\cdot}4$ per century; they show that the eclipses adduced by Mr. Cowell do not suffice to establish it as he supposed, and are quite consistent with systems of data very different from that adopted by him, and which do not involve the difficulties inseparable from his. But it must be distinctly understood that this is all. In many respects the preceding examination of the ancient eclipses strengthens Mr. Cowell's view, for it shows that his data serve to represent other eclipses besides those considered by him, and in particular that they serve to represent the eclipse of Larissa, the most critically important of them all. Indications of the existence of such a secular term, though smaller in amount, have been found cropping up repeatedly in the discussion of the early observations; but it is very difficult to differentiate such a secular term from the very similar effects which would be produced by inequalities of very long period in the complete expression for the Moon's latitude, and there is some reason for supposing these to exist.

A much wider question is raised by Mr. Cowell in his second paper, that the apparent secular acceleration of $+4''{\cdot}4$ in the angular distance of the Moon from its node really arises from the existence of a secular acceleration of $+4''{\cdot}1$ in the mean longitude of the Earth.

Assuming that the ancient eclipses of the Sun show that there exist a secular acceleration of $+4''{\cdot}4$ in the angular distance of the Moon from the node and a secular acceleration of $+6''{\cdot}8$ in the angular distance of the Moon from the Sun, Mr. Cowell points out that this can be represented by supposing the secular acceleration of the Moon's node to have the theoretical value of $+6''{\cdot}5$, and assigning a secular acceleration of $6''{\cdot}8+4''{\cdot}1=10''{\cdot}9$ to the mean longitude of the Moon, and of $+4''{\cdot}1$ to the apparent mean longitude of the Sun.

The effect of this change of origin of the secular acceleration leaves unchanged the epoch of conjunction of the mean longitudes of the Sun and Moon, and does not alter the values of either $D=(L-L')$ or $F=(L-☊)$; hence Mr. Cowell concludes that it leaves unchanged the figures of his eclipse calculations, except perhaps a small change of less than $20''$ in the correction for parallax, which he regards as quite unimportant.*

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* This is an oversight of Mr. Cowell's. It is true that neither D nor F is altered; but both g and g' are changed by quantities which may exceed a degree of arc. Hence both V and V′ and their difference $V-V'$ may be changed by considerable quantities, possibly amounting to several hundreds of seconds of arc in the case of the earlier eclipses; whilst U will also be

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But it will be seen that this new hypothesis as to the origin of the correction required by the Moon's argument of latitude removes one theoretical difficulty only at the expense of restoring another; for though the assumed secular acceleration of the Moon's node is brought into accord with theory, it is only at the cost of reinstating the discordance between 6″·0, the theoretical value of the secular acceleration in mean longitude, and that of 10″·9, the value necessitated by Mr. Cowell's new hypothesis. It is true that such a discordance conceivably might arise from tides in the body of the Earth, regarded as a thin rigid shell covering a viscous fluid interior, or in an ocean entirely covering the surface of the Earth, though not from tides in smaller oceans having the physical configuration of those actually existing on the terrestrial surface; but there is no real evidence in support of a tidal retardation in the rotation of the Earth of a nature which could be held to account for a material discordance between the observed value of the secular acceleration of the Moon's mean longitude and that derived by theory from the diminution in the eccentricity of the Earth's orbit.

The effect of this amended hypothesis for bringing the lunar tables into accord with the ancient eclipses by assuming the existence of a considerable secular acceleration in the motion of the Earth from the effects of a resisting medium in space proceeds far beyond the effect on the eclipses, but, except in respect of its effects on the observations of the transits of *Mercury*, these have not been considered by Mr. Cowell. It is not proposed, however, to discuss the matter beyond its effects on the apparent longitudes of the Sun and Moon, though these form a very small portion of the difficulties which would have to be overcome in reconciling with observation the effects due to this new hypothesis of Mr. Cowell's. The assumed secular retardation cannot be supposed to be confined merely to the Earth; so the assumed law of resistance in space must be such that, despite the great physical differences between the different planets, the apparent secular accelerations of the other planets would differ so slightly from the apparent secular acceleration of the Earth that no appreciable signs of the difference would be apparent. The more important cases would be those of *Venus* and *Mars*, whose observed secular variations are closely in accord with those derived from theory, and Mr. Cowell would have some difficulty in finding a law of resistance in space which would leave the relative motions of the three planets unaffected. The effects of the resisting medium would extend, of course, to the eccentricities as well as to the mean distances.

altered, though to a smaller degree. Hence Mr. Cowell's eclipse calculations will no longer apply, and must be revised to meet the effects of the changed hypotheses. In the case of the five particular eclipses considered by Mr. Cowell it is probable that small alterations in his data will suffice to bring them into accord with the changed conditions, and so leave his conclusions unaffected.

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Restricting the consideration to the effects on the mean longitudes of the Sun and Moon, assuming 1900 as the epoch, the effect of the new secular term would be to increase the tabular longitudes by a second of arc in 1850, so it will be necessary to increase the secular mean motions by some two seconds of arc, in order to bring the new tabular places into the same accord with observation as exists with the present tables as revised. The effect will be to increase the tabular longitudes of the Sun and Moon at different epochs by the quantities given in column I. of the table which follows. But it will be seen that this leaves discordances of over 6″ between Bradley's observations of 1750–60 and the tables as altered, discordances far greater than are admissible; hence to bring these earlier observations into accord with the changed tables a still further increase must be made in the secular mean motion, with the consequence that if the ancient eclipses are still to be represented, a further increase must also be made in the value of the new secular acceleration. Adopting +5″·5 for the centennial increase in the mean longitude and +4″·33 for the coefficient of the new secular term, the tabular places of the Sun and Moon at the different epochs will be changed by the quantities shown in Column II. For comparison, the columns headed "Sun" and "Moon" show the outstanding discordances between the observed and latest revised tabular places of the Sun and Moon, and the columns "Sun + II." and "Moon + II." the residuals which will remain outstanding when compared with tables modified by the quantities given under Column II.

Epoch.	I.	II.	Sun.	Sun+II.	Moon.	Moon+II.
	″	″	″	″	″	″
1900	+ ·00	+ ·00	+ ·20	+ ·20	+ ·55	+ ·55
1880	− ·24	− ·93	− ·07	− 1·00	+ ·12	− ·81
1860	− ·14	− 1·51	− ·41	− 1·92	− ·04	− 1·55
1840	+ ·28	− 1·75	− ·45	− 2·20	− ·21	− 1·96
1820	+ 1·02	− 1·63	+ 1·11	− ·52	+ ·52	− 1·11
1800	+ 2·10	− 1·17	− ·72	− 1·89	− ·75	− 1·92
1780	+ 3·50	− ·37	− ·95	− 1·32	+ ·79	+ ·42
1760	+ 5·24	+ ·78	+ ·22	+ 1·00	− 1·70	− ·92
1740	+ 7·30	+ 2·28	+ ·30	+ 2·58	− 1·94	+ ·34
1720	+ 9·68	+ 4·12	...	...	− ·13	+ 3·99
1700	+ 12·44	+ 6·65	...	...	− ·39	+ 6·26
1680	+ 15·44	+ 8·85	...	...	+ 1·91	+ 10·74
1650	+ 18·81	+ 11·74	...	...	+ ·98	+ 12·72

The resulting discordances between the tabular and observed places of the Moon between 1740 and 1900 could be eliminated by an increase in the assumed coefficients of the terms of very long period arising from the perturbations by the planets, but it

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is doubtful if the very large discordances between the observed and tabular times of the numerous occultations of stars observed between 1680 and 1730 could be similarly eliminated. But there is no such means of removing the discordances which the change would introduce between the observed and tabular places of the Sun during the period 1750–1880, and these discordances are far greater than can be ascribed to the outstanding errors of observation.

Natal Observatory:
1906 *April* 3.

On the Ratios of the triangles in the Determination of the Elliptic Orbit from three Observations. By Shin Hirayama.

1. In this paper I determine the ratios of the triangles which play an important part in the problem of determining the orbit from three observations. Gibbs has determined these ratios by expanding the heliocentric rectangular coordinates of the planet in powers of time, while I expand them in a series of sines and cosines of the mean anomaly and in powers of eccentricity, as is generally done in the elliptic theory.

2. For the three observations of the planet at the times t_1, t_2, and t_3 we have :

$$t_3 - t_2 = \tau_1. \qquad t_3 - t_1 = \tau_2. \qquad t_2 - t_1 = \tau_3.$$

x_1, y_1, z_1 x_2, y_2, z_2 x_3, y_3, z_3	three rectangular heliocentric coordinates of planet.
r_1, r_2, r_3	radii vectores of planet.
n	mean daily motion expressed in seconds of arc.
a	semi-major axis of planet.

3. Let

$$x = a_0 + a_1 \sin nt + b_1 \cos nt + a_2 \sin 2nt + b_2 \cos 2nt \quad . \quad . \quad (1)$$

where a_0, a_1, b_1, a_2, b_2, are five constants depending on the elements of the orbit. Then the accelerations along the x axis corresponding to the three positions will be represented by the following expressions respectively :

$$\textbf{For } t = -\tau_3\,; \quad \frac{d^2x}{dt^2} = -\frac{x_1}{r_1^3} = -n^2a_1 \sin n\tau_3 + n^2b_1 \cos n\tau_3 - 4n^2a_2 \sin 2n\tau_3 + 4n^2b_2 \cos 2$$

$$\textbf{For } t = 0 \quad ; \quad \frac{d^2x}{dt^2} = -\frac{x_2}{r_2^3} = \qquad n^2b_1 \qquad + 4n^2b_2$$

$$\textbf{For } t = +\tau_1\,; \quad \frac{d^2x}{dt^2} = -\frac{x_3}{r_3^3} = +n^2a_1 \sin n\tau_1 + n^2b_1 \cos n\tau_1 + 4n^2a_2 \sin 2n\tau_1 + 4n^2b_2 \cos 2n$$